Clasa a IX-a ➡ Barem de corectare

Problema 1

$\frac{x_1^2}{a_1}+\frac{x_2^2}{a_2}+\frac{x_3^2}{a_3}+\cdots+\frac{x_{2026}^2}{a_{2026}} \geq \frac{(x_1+x_2+x_3+\cdots+x_{2026})^2}{a_1+a_2+a_3+\cdots+a_{2026}}$	2p
$(x_1+x_2+x_3+\cdots+x_{2026})^2 = [x_1 \cdot (1+q^2+q^3+\cdots+q^{2025})]^2$	1p
$a_1+a_2+a_3+\cdots+a_{2026} = a_1 \cdot \left(1+\frac{1}{q}+\frac{1}{q^2}+\cdots+\frac{1}{q^{2026}}\right)$	1p
$\frac{x_1^2 \cdot (1+q^2+q^3+\cdots+q^{2025})^2}{a_1 \cdot (1+q^2+q^3+\cdots+q^{2025})} \cdot q^{2025} =$	1p
$= \frac{x_1^2 \cdot (1+q^2+q^3+\cdots+q^{2025})}{x_{2026}} \cdot q^{2025} =$	1p
$= \frac{x_1^2 \cdot (1+q^2+q^3+\cdots+q^{2025})}{x_1 \cdot q^{2025}} \cdot q^{2025} =$	1p
$= x_1 \cdot (1+q^2+q^3+\cdots+q^{2025}) =$	1p
$= x_1+x_2+x_3+\cdots+x_{2026}$	1p

Problema 2: Fie f:R→R cu proprietatea $f(f(x)) = x^2+\frac{1}{4}$, $\forall$ x $\in$ R. Determinați $f\left(\frac{1}{2}\right)$.

$f(f(f(x))) = f\left(x^2+\frac{1}{4}\right)$	2p
$f(f(f(x))) = f^2(x)+\frac{1}{4}$	2p
$f^2(x)+\frac{1}{4} = f\left(x^2+\frac{1}{4}\right)$	1p
$x = \frac{1}{2} \Rightarrow f^2\left(\frac{1}{2}\right)+\frac{1}{4} = f\left(\left(\frac{1}{2}\right)^2+\frac{1}{4}\right)$	1p
$f^2\left(\frac{1}{2}\right)+\frac{1}{4} = f\left(\frac{1}{2}\right)$	1p
$\left(f\left(\frac{1}{2}\right)-\frac{1}{2}\right)^2 = 0$	1p
$f\left(\frac{1}{2}\right) = \frac{1}{2}$	1p

Problema 3

Fie patrulaterul convex ABCD, iar G_1 și G_2 centrele de greutate ale triunghiurilor ACD și BCD. Demonstrați că $\overrightarrow{AG_1}-\overrightarrow{BG_2} = 2 \cdot \overrightarrow{G_1G_2}$.

$3 \cdot \overrightarrow{OG_1} = \overrightarrow{OA}+\overrightarrow{OC}+\overrightarrow{OD}$	1p
$3 \cdot \overrightarrow{OG_2} = \overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$	1p
$3 \cdot (\overrightarrow{OG_2}-\overrightarrow{OG_1}) = \overrightarrow{OB}-\overrightarrow{OA}$	2p
$3 \cdot \overrightarrow{G_1G_2} = \overrightarrow{AB}$	2p
$G_1G_2 \parallel AB$ și $\frac{G_1G_2}{AB} = \frac{1}{3}$	3p